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Absorption spectrum of a resonantly driven degenerate V-type atom with a dc-field coupling between excited states

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Abstract. We study the absorption spectra of a degenerate V-type atom, where a resonant driving field and a probe field drive different branches of transitions and a dc field is applied to drive the transition between two excited states. The effects of vacuum induced coherence (VIC) on the absorption spectra are investigated. It is demonstrated that in some special cases the VIC can lead to the depression of absorption and narrow resonance. The origin of these features are discussed. When the pump field and the dc field have the same intensity, it is interesting to find that the whole absorption spectrum comes mainly from the absorptions induced by the interferences among different transitions between dressed states.

PACS. 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light; electromagnetically induced transparency and absorption – 42.50.Hz Strong-field excitation of optical transitions in quantum systems; multiphoton processes; dynamic Stark shift

1 Introduction

The properties of an atomic system can be changed significantly when it is driven by a strong, resonant, coherent field. Mollow first studied the physical properties of a strongly driven two-level atomic system and discovered new features in the emission spectrum [1]. These features can be best understood through the dressed-atom model of the system [2]. Mollow, in a later paper [3], calculated the absorption spectrum of a weak probe field by a two-level atom driven by an intense pump field and demonstrated the possibility of amplification of the probe field. This gain can also be understood in terms of dressed states [4]. Autler-Townes effect, in which the probe field detect the transitions from one of the two states connected by the pump field to the third state, leading to two absorption resonances, is also related to dressed states. Various experiments have confirmed the presence of such effect [5,6].

In recent years, absorption, dispersion and spontaneous emission processes subject to the quantum interference among different decay channels have been of considerable interests. In most of early works, the usual denominations of this new type of quantum interference include vacuum induced coherence (VIC) and spontaneous generated coherence (SGC). For definiteness, in our paper we use the former denomination, namely, VIC. The VIC can lead to many remarkable phenomena, such as fluorescence quenching and spectral line narrowing [7], phase-dependent line shapes [8], constant population of the dark state [9], etc. In the studying of the spectra of a basic V system to a probe field, the VIC is also responsible for many novel effects. Zhou et al. considered the absorption spectrum of a V system in which the probe field detect both the two branches of transitions. It is demonstrated that some effects, such as narrow resonances, transparency and emission without population inversion, have their origin in the phenomenon of the VIC [10]. Dong et al. studied the absorption properties of a driven V-type system. It is found that difference strengths of the VIC can result in different line shapes for emission peaks and absorption peaks [11]. In another scheme, where pump field and probe field drive separately different branches of transitions in a V system. It is shown that the VIC leads to emission feature instead of the usual absorption feature in the Autler-Townes doublet [12].

In this paper, we study the absorption spectrum of a degenerate V-type system. In this system the two upper levels are coupled by a dc field. One of the dipole transition of the atom is driven by a pump field while another perturbed by a probe field. This type of system can be found in real atoms, which has been suggested by Ficek [13], so the experiments can be done without any rigorous restrictions.

The paper is organized as follows: in Section 2, we describe the model and derive the basic density-matrix equations. In Section 3, we present the numerical results of the steady-state absorption spectra graphically. In Section 4, the absorption spectra are investigated in detail through the dressed-atom model. In Section 5, we give a brief summary.

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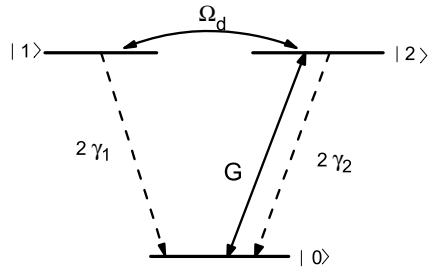

Fig. 1. Three-level V-type system with a pump laser field G and a dc field Ω_d.

2 Theoretical model and density-matrix equations

Consider a three-level system in the V configuration as shown in Figure 1. The atom consists of two degenerate excited levels $|1\rangle$ and $|2\rangle$ separated from the ground level $|0\rangle$ by transition frequency ω, and connected by the electric dipole moments $\boldsymbol{d}_{10}$ and $\boldsymbol{d}_{20}$, respectively. A resonant pump field of amplitude $\boldsymbol{E}_0$ and Rabi frequency $G = \boldsymbol{d}_{20} \cdot \boldsymbol{E}_0/\hbar$ is applied to the optical transition $|0\rangle \leftrightarrow |2\rangle$. Levels $|1\rangle$ and $|2\rangle$ are coupled by a dc field with Rabi frequency Ω_d.

In the frame rotating with the pump field frequency ω the master equation is of the form

$$\dot{\rho} = -i/\hbar[H, \rho] + \mathcal{L}\rho, \tag{1}$$

where the Hamiltonian is

$$-\hbar\Omega_d(\sigma_{12} + \sigma_{21}) - \hbar G(\sigma_{02} + \sigma_{20}) \tag{2}$$

and the damping term is

$$\begin{aligned}\mathcal{L}\rho = &-\gamma_1\sigma_{10}\sigma_{01}\rho - \gamma\sigma_{20}\sigma_{01}\rho - \gamma_2\sigma_{20}\sigma_{02}\rho - \gamma\sigma_{10}\sigma_{02}\rho \\ &+2\gamma_1\sigma_{01}\rho\ \sigma_{10} + 2\gamma\sigma_{01}\rho\sigma_{20} + 2\gamma\sigma_{02}\rho\sigma_{10} \\ &+2\gamma_2\sigma_{02}\rho\sigma_{20} - \gamma_1\rho\sigma_{10}\sigma_{01} - \gamma\rho\sigma_{20}\sigma_{01} \\ &-\gamma\rho\sigma_{10}\sigma_{02} - \gamma_2\rho\sigma_{20}\sigma_{02}.\end{aligned} \tag{3}$$

In equations (2, 3), $\sigma_{mn} \equiv |m\rangle\langle n|$ $(m, n = 0, 1, 2)$ is the transition operator, γ_i is the spontaneous decay constant of the excited state $|i\rangle$ $(i = 1, 2)$ to the ground level $|0\rangle$,

$$\gamma = \sqrt{\gamma_i\gamma_j}p \quad (i \neq j = 1, 2) \tag{4}$$

represent the VIC effect arising from the cross-coupling between two decay paths $|1\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |0\rangle$. The parameter p is defined as $p = \boldsymbol{d}_{i0} \cdot \boldsymbol{d}_{j0}/|\boldsymbol{d}_{i0}||\boldsymbol{d}_{j0}| = \cos\theta$ where θ is the angle between the two dipole moments $\boldsymbol{d}_{10}$ and $\boldsymbol{d}_{20}$. In this model we assume that the pump field acts only on the transition $|0\rangle \leftrightarrow |2\rangle$, i.e., $\boldsymbol{d}_{10} \cdot \boldsymbol{E}_0 = 0$. Then, the relation between the normalized pump field and the parameter p is obtained as:

$$G = \frac{|\boldsymbol{d}_{20}||\boldsymbol{E}_0|}{\hbar}\sqrt{1-p^2} = G_0\sqrt{1-p^2}. \tag{5}$$

According to the master equation in equation (1), the equations of motion of the density matrix elements take the form

$$\begin{aligned}\dot{\rho}_{11} &= i\Omega_d(\rho_{21} - \rho_{12}) - 2\gamma_1\rho_{11} - \gamma(\rho_{21} + \rho_{12}), \\ \dot{\rho}_{00} &= iG(\rho_{20} - \rho_{02}) + 2\gamma_1\rho_{11} + 2\gamma(\rho_{12} + \rho_{21}) + 2\gamma_2\rho_{22}, \\ \dot{\rho}_{12} &= -iG\rho_{10} + i\Omega_d(\rho_{22} - \rho_{11}) - \gamma_1\rho_{12} - \gamma\rho_{22} - \gamma\rho_{11} \\ &\quad -\gamma_2\rho_{12}, \\ \dot{\rho}_{10} &= -iG\rho_{12} + i\Omega_d\rho_{20} - \gamma_1\rho_{10} - \gamma\rho_{20}, \\ \dot{\rho}_{20} &= iG(\rho_{00} - \rho_{22}) + i\Omega_d\rho_{10} - \gamma\rho_{10} - \gamma_2\rho_{20}.\end{aligned} \tag{6}$$

The density-matrix elements obey the conditions $\rho_{00} + \rho_{11} + \rho_{22} = 1$ and $\rho_{IJ} = \rho^*_{JI}$, with $I, J = 0, 1, 2$.

3 Absorption spectra

We assume that the atomic system is illuminated by a weak, frequency-tunable probe beam. Linear response theory gives the steady-state probe absorption spectrum to be

$$A(\Delta_p) = \Re\int_0^\infty \lim_{t\to\infty}\langle[D(t+\tau), D^+(t)]\rangle e^{i\Delta_p\tau}d\tau, \tag{7}$$

where, $D(t) = d_{10}\sigma_{01}$ is the component of the atomic polarization operator in the direction of the probe field polarization vector $\boldsymbol{e}_p$, with $d_{10} = \boldsymbol{d}_{10} \cdot \boldsymbol{e}_p$ being the dipole moment of the atomic transition from $|0\rangle$ to $|1\rangle$, Δ_p is the detuning between the probe frequency and the atomic transition frequency ω. The absorption spectrum defined in equation (7) can be calculated with the help of the quantum regression theorem and Bloch equations (6). Here the detailed processes of the calculation are not shown, which can be found in other previous papers [14,15].

In what follows we present our numerical results by plotting the absorption spectra $A(\Delta_p)$.

In Figure 2, for $G_0 = 0, \Omega_d = 3$, we plot the absorption spectra for different degrees of the VIC. Figures 2a, 2b and 2c are for $p = 0$, 0.5 and 0.99, respectively. From Figure 2a one can see the spectrum exhibits two absorption lines symmetrically in the absence of the VIC. When $p = 0.5$, representing an obvious effect of the quantum interference, the magnitude of the left absorption resonance is reduced while the linewidth broadened. For the right resonance, the effects are just opposite. When $p = 0.99$, as shown in Figure 2c, the linewidth of the right absorption line decreases to a very little value which indicate a narrow resonance. Moreover, the amplitude of the left absorption resonance is depressed to a very low value (about 0.251). By further investigation we found that the limit value of the amplitude of the left resonance is 0.25 when p approaches its maximum value ($p = 1$). Compared with the right one, the absorption for left resonance can be neglected rationally which indicate an induced transparency effect for the probe field in transition $|A\rangle \rightarrow |B\rangle$. These phenomena are apparently results of the quantum interference (VIC) and will be explained in the following section.

The absorption spectra for the case of $G_0 = 3$ are depicted in Figure 3, where the other parameters are the

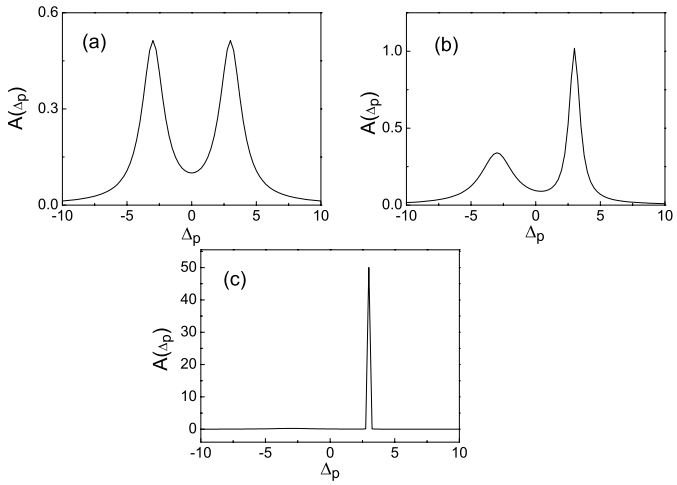

Fig. 2. Absorption spectra as a function of probe detuning Δ_p for various p. The parameters employed are $\gamma_1 = \gamma_2 = 1$; $\Omega_d = 3, G_0 = 0$. (a) $p = 0$; (b) $p = 0.5$; (c) $p = 0.99$.

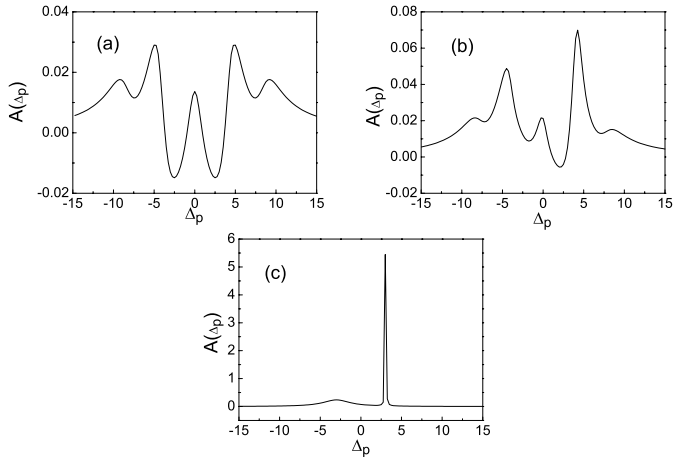

Fig. 3. Absorption spectra as a function of probe detuning Δ_p for various p. The parameters employed are $\gamma_1 = \gamma_2 = 1$; $\Omega_d = 3, G_0 = 3$. (a) $p = 0$; (b) $p = 0.5$; (c) $p = 0.99$.

same as those used in Figure 2. From Figure 3a, we see the absorption spectrum shows a multi-peak structure. In Section 4 we will prove that this spectral feature interestingly derive mostly from the interferences of the different absorption processes between dressed states of the system. In Figure 3c, for $p = 0.99$, it is seen that the spectrum resemble the profile shown in Figure 2c, but with different magnitudes.

Figure 4 shows the absorption spectra for $G_0 = 25$ and different degrees of the VIC. Figures 4a, 4b and 4c are for $p = 0$, 0.45 and 0.9, respectively. In Figure 4a, one see that the absorption spectrum shows two symmetric absorption lines. When $p = 0.45$, the spectrum is unsymmetrical where the magnitude of the absorption resonance leftward is enhanced and that rightward reduced. Moreover, the absolute values of the detunings of two resonances are decreased. Consider a higher degree of the VIC, the effects mentioned above strengthen, which can be seen in Figure 4c.

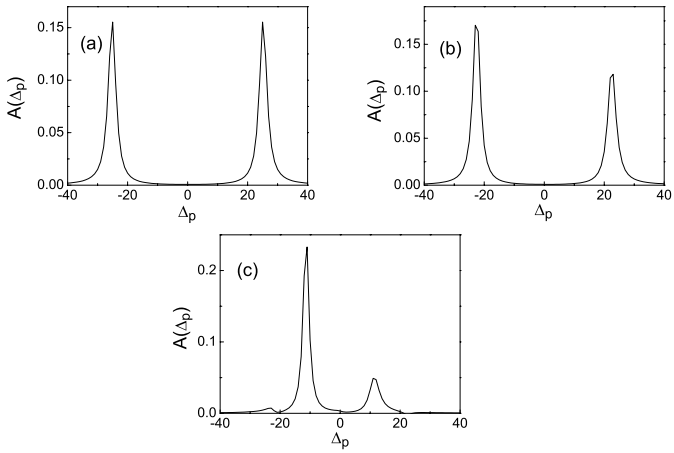

Fig. 4. Absorption spectra as a function of probe detuning Δ_p for various p. The atomic parameters $\gamma_1 = \gamma_2 = 1$; $\Omega = 3, G_0 = 25$. (a) $p = 0$; (b) $p = 0.45$; (c) $p = 0.9$.

4 Dressed-state explanations

We present the physical interpretations of the spectral features shown in Section 3 by the dressed-atom models of the system under different parameter settings.

The Hamiltonian for the atom and interacting with the dc field and the quantized pump field can be written as

$$H_c = \hbar\omega a^+ a + \hbar\omega(\sigma_{11} + \sigma_{22}) + \hbar g(\sigma_{02}a^+ + \sigma_{20}a) - \hbar\Omega_d(\sigma_{12} + \sigma_{21}). \quad (8)$$

In equation (8) a and a^+ are the annihilation and creation operators for the pump field and g is the coupling constants between the atom and the pump field. Rabi frequency of the pump field in the quantum form is defined as

$$-\hbar g\sqrt{\bar{N}+1} = \hbar G, \quad (9)$$

where $\bar{N}$ is the average photon number of the pump field. The dressed states, as defined by the eigenvalue equation, $H_c|\alpha\rangle = E_\alpha|\alpha\rangle$, are of the form

$$\begin{aligned} |A(N,\Omega_d)\rangle &= -c|0,N+1\rangle + b|1,N\rangle, \\ |B(N,\Omega_d)\rangle &= (b/\sqrt{2})|0,N+1\rangle + (c/\sqrt{2})|1,N\rangle \\ &\quad +(1/\sqrt{2})|2,N\rangle, \\ |C(N,\Omega_d)\rangle &= -(b/\sqrt{2})|0,N+1\rangle - (c/\sqrt{2})|1,N\rangle \\ &\quad +(1/\sqrt{2})|2,N\rangle, \end{aligned} \quad (10)$$

with energies

$$\begin{aligned} E_A &= (N+1)\hbar\omega, \\ E_B &= (N+1)\hbar\omega - \hbar\Omega, \\ E_C &= (N+1)\hbar\omega + \hbar\Omega, \end{aligned} \quad (11)$$

where N represents the photon number of the pump field, and

$$\Omega = \sqrt{G^2 + \Omega_d^2}, \quad b = G/\Omega, \quad c = \Omega_d/\Omega. \quad (12)$$

The dressed states defined by equation (10) form an infinite ladder of manifolds composed of three states with N varies. Adjacent manifolds are separated by $\hbar\omega$. Inside a manifold, the states $|C(N,\Omega_d)\rangle$ and $|B(N,\Omega_d)\rangle$ are separated by $\hbar\Omega$ and $-\hbar\Omega$ from the state $|A(N,\Omega_d)\rangle$, respectively. It is worth noting that the steady-state populations of the atom in the dressed states can be achieved by solving numerically the Bloch equations in the dressed-state picture (given in the Appendix). For different manifolds, the populations of states $|A(N,\Omega_d)\rangle$, $|B(N,\Omega_d)\rangle$ and $|C(N,\Omega_d)\rangle$ vary negligibly with N

$$\begin{aligned}(\text{i.e., } &\rho_{A(N,\Omega_d)A(N,\Omega_d)} \approx \rho_{A(N+1,\Omega_d)A(N+1,\Omega_d)},\\ &\rho_{B(N,\Omega_d)B(N,\Omega_d)} \approx \rho_{B(N+1,\Omega_d)B(N+1,\Omega_d)},\\ &\rho_{C(N,\Omega_d)C(N,\Omega_d)} \approx \rho_{C(N+1,\Omega_d)C(N+1,\Omega_d)})\ [16].\end{aligned}$$

In the following, we present our explanations of the absorption spectra in Figures 2, 3 and 4.

(1) $G_0 = 0$, $\Omega_d = 3$. In the absence of the pump field the eigenstates and energies can be obtained by setting $b = 0$ in equations (10, 11) and exclude the number states of the pump field, which reads

$$\begin{aligned}|A(\Omega_d)\rangle &= -|0\rangle,\\ |B(\Omega_d)\rangle &= (1/\sqrt{2})|1\rangle + (1/\sqrt{2})|2\rangle,\\ |C(\Omega_d)\rangle &= -(1/\sqrt{2})|1\rangle + (1/\sqrt{2})|2\rangle,\end{aligned} \tag{13}$$

with energies

$$E_A = \hbar\omega,\ E_B = \hbar\omega - \hbar\Omega_d,\ E_C = \hbar\omega + \hbar\Omega_d. \tag{14}$$

The probe photon absorption occurs in the transitions of dressed states $|A\rangle \to |C\rangle$ with frequency $\omega + \Omega_d$ and $|A\rangle \to |B\rangle$ with frequency $\omega - \Omega_d$. These two frequencies correspond to the detuning values of the two peaks in Figure 2 locating at $\Delta_1 = \pm\Omega_d$.

The new effects exhibited when the VIC is taken into account can be explained through calculating the absorption spectrum in the dressed-state picture. Replacing $D(t)$ in equation (7) with the component of the atomic polarization operator in the direction of the probe field polarization in the dressed-state picture, which reads

$$D = d_{CA}\sigma_{AC} + d_{BA}\sigma_{AB}, \tag{15}$$

where $d_{CA} = \boldsymbol{d}_{CA}\cdot\boldsymbol{e}_p$ and $d_{BA} = \boldsymbol{d}_{BA}\cdot\boldsymbol{e}_p$. Employing the the Bloch equations in the dressed-state picture, the analytic form of the absorption spectrum is given as

$$\begin{aligned}A(\Delta_p) = \frac{1}{2}\mathrm{Re}\Bigg[&\frac{[1-p-i(\Delta_p-\Omega_d)](\bar\rho_{AA}-\bar\rho_{BB})}{[1+p-i(\Delta_p+\Omega_d)][1-p-i(\Delta_p-\Omega_d)]}\\ &+\frac{[1+p-i(\Delta_p+\Omega_d)](\bar\rho_{AA}-\bar\rho_{CC})}{[1+p-i(\Delta_p+\Omega_d)][1-p-i(\Delta_p-\Omega_d)]}\\ &+\frac{(i\Omega_d - i\Delta_p - p)\bar\rho_{BC} + (1+p-i\Omega_d - i\Delta_p)\bar\rho_{CB}}{[1+p-i(\Delta_p+\Omega_d)][1-p-i(\Delta_p-\Omega_d)]}\\ &+\frac{\bar\rho_{BB}-\bar\rho_{AA}}{[1+p-i(\Delta_p+\Omega_d)][1-p-i(\Delta_p-\Omega_d)]}\Bigg],\end{aligned} \tag{16}$$

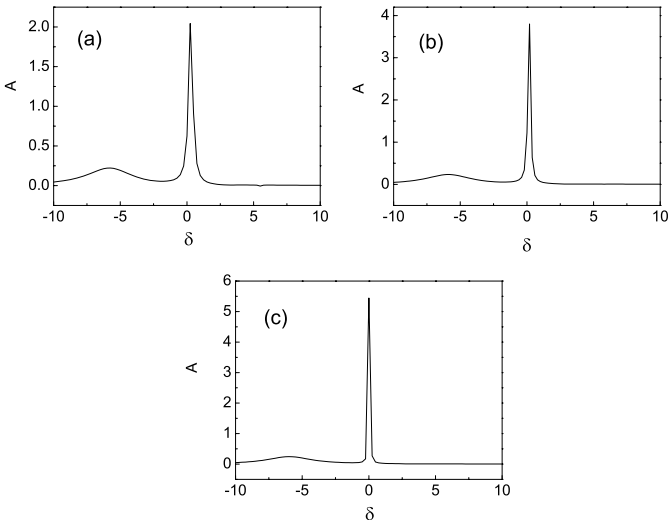

Fig. 5. Absorption spectra for fixed probe detuning ($\Delta_p = \Omega_d$) as a function of pump detuning δ for various p. The atomic parameters $\gamma_1 = \gamma_2 = 1$; $\Omega_d = 3, G_0 = 3$. (a) $p = 0.93$; (b) $p = 0.96$; (c) $p = 0.99$.

where $\bar\rho_{ij}(i,j = A,B,C)$ represents the steady-state value of the matrix elements of the density operator. It should be added that in obtaining equation (16) we have made use of the values of parameters γ_1 and γ_2 in Figure 2. Expression (16) can be simplified by replacing $\bar\rho_{ij}$ with the corresponding values, which are calculated as $\bar\rho_{AA} = 1$, $\bar\rho_{BB} = \bar\rho_{AB} = \bar\rho_{AC} = \bar\rho_{BC} = 0$. In this situation, the expression reduces to

$$A(\Delta_p) = \frac{1}{2}\mathrm{Re}\left[\frac{1-2i\Delta_p}{[1+p-i(\Delta_p+\Omega_d)][1-p-i(\Delta_p-\Omega_d)]}\right]. \tag{17}$$

The spectra depicted according to equation (17) gives the exactly equal spectral profiles shown in Figure 2. It is clear that the spectrum has two absorption resonances at $\Delta_p = \Omega_d$ and $\Delta_p = -\Omega_d$. The VIC acts to broaden the linewith of the resonance at $\Delta_p = -\Omega_d$ and reduce the linewidth at $\Delta_p = \Omega_d$. The magnitudes of the two resonances can be obtained by setting $\Delta_p = \Omega_d$ and $-\Omega_d$, which are

$$\begin{aligned}A(\Delta_p = \Omega_d) &= \frac{1}{2(1-p)}\left[1 - \frac{(1+p)p}{(1+p)^2 + 4{\Omega_d}^2}\right],\\ A(\Delta_p = -\Omega_d) &= \frac{1}{2(1+p)}\left[1 + \frac{(1-p)p}{(1-p)^2 + 4{\Omega_d}^2}\right],\end{aligned} \tag{18}$$

while when p increases, $A(\Delta_p = \Omega_d)$ increases and $A(\Delta_p = -\Omega_d)$ decreases. These conclusions agree with the numerical results shown in Figure 2.

(2) $G_0 = 3$, $\Omega_d = 3$. In this case the eigenstates of the system and eigenvalues are exact the expressions given in equations (10, 11). The probe field will detect all the nine transitions between the dressed states of the two neighboring manifolds, as shown in Figure 6.

We still calculate the absorption spectrum in the dressed-state picture with the parameter values employed

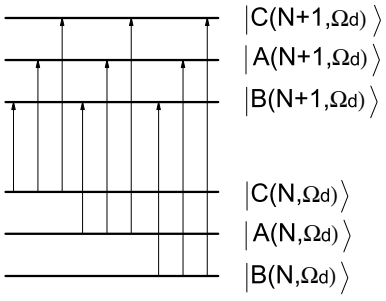

Fig. 6. Detected transitions by the probe field between dressed states of two neighboring manifolds. The parameters employed are the same as those in Figure 3a.

in Figure 3. For the simplicity of denotation, we label $|A(N+1,\Omega_d)\rangle$, $|B(N+1,\Omega_d)\rangle$, $|C(N+1,\Omega_d)\rangle$ with $|A'\rangle$, $|B'\rangle$, $|C'\rangle$, and $|A(N,\Omega_d)\rangle$, $|B(N,\Omega_d)\rangle$, $|C(N,\Omega_d)\rangle$ with $|A\rangle$, $|B\rangle$ and $|C\rangle$. The component of the atomic polarization operator in the dressed-state picture in the direction of the probe field polarization can be written in the form

$$\begin{aligned} D_d(t) = {} & d_{A'A}\sigma_{AA'}(t) + d_{A'B}\sigma_{BA'}(t) + d_{A'C}\sigma_{CA'}(t) \\ & + d_{B'A}\sigma_{AB'}(t) + d_{B'B}\sigma_{BB'}(t) + d_{B'C}\sigma_{CB'}(t) \\ & + d_{C'A}\sigma_{AC'}(t) + d_{C'B}\sigma_{BC'}(t) + d_{C'C}\sigma_{CC'}(t). \end{aligned} \tag{19}$$

By substituting equation (19) with $t \to t+\tau$ into equation (7), we obtain the expression of the absorption spectrum in the dressed-state picture

$$\begin{aligned} A_d(\Delta_p) &= \Re \int_0^\infty \lim_{t\to\infty} \langle [D_d(t+\tau), D_d^+(t)] \rangle e^{i\Delta_p \tau} d\tau \\ &= \Re \sum_{ijmn} d_{ij} d_{mn} \\ &\quad \times \int_0^\infty \lim_{t\to\infty} \langle [\sigma_{ij}(t+\tau), \sigma_{mn}(t)] \rangle e^{i\Delta_p \tau} d\tau \\ &= \sum_{ijmn} A^d_{ijmn}(\Delta_p) \\ &\quad (j,m = A', B', C' \text{ and } i,n = A, B, C), \end{aligned} \tag{20}$$

where $d_{kl} = \boldsymbol{d}_{kl} \cdot \boldsymbol{e}_p$ ($k = A', B', C'$, and $l = A, B, C$). The summation in equation (20) consists of 81 terms of contributions, which we can divide into two types: (I) terms directly related with the transitions between dressed states, e.g., $A^d_{BA'A'B}$, which is associated with the absorption of probe field in the transition $|B\rangle \to |A'\rangle$. There are totally 9 terms belong to type I, corresponding to the absorptions in nine transitions between dressed states. (II) terms related with the interference between two different transitions, e.g., $A^d_{BA'B'C}$, which indicate the absorption induced by the interference of two transitions $|B\rangle \to |A'\rangle$ and $|C\rangle \to |B'\rangle$. In the following, we will investigate the contributions of the terms of the two types to the whole spectrum. It is worth adding that the steady-state populations of the dressed states under the parameters used in Figure 3a are calculated as: $\bar{\rho}_{AA} = 0.6014, \bar{\rho}_{BB} \approx \bar{\rho}_{CC} = 0.1993$.

In Figure 7 we plot the contributions of partial terms of type I. The solid line and the dashed line shown in Figure 7a correspond to the absorption spectrum of the probe field in the transitions $|A\rangle \to |i\rangle$

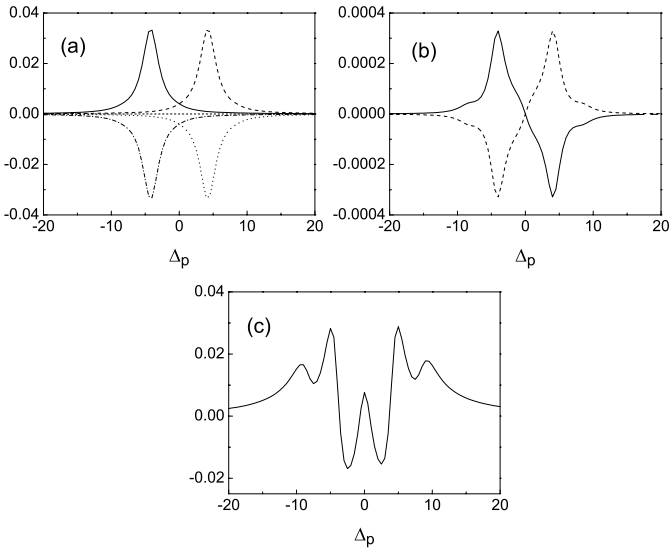

Fig. 7. Different terms of the absorption spectrum. (a) $A^d_{AB'B'A}(\Delta_p)$ (solid line), $A^d_{AC'C'A}(\Delta_p)$ (dashed line), $A^d_{BA'A'B}(\Delta_p)$ (dotted line), $A^d_{CA'A'C}(\Delta_p)$ (dashed-dotted line), the superposition of the terms above (short dashed line). (b) $A^d_{BB'B'B}(\Delta_p)$ (solid line), $A^d_{CC'C'C}(\Delta_p)$ (dashed line). (c) Superposition of the terms which belong to type II. The parameters employed are the same as those in Figure 3a.

$$\text{(given by } (d_{iA})^2 \text{Re} \int_0^\infty \lim_{t\to\infty} \langle [\sigma_{Ai}(t+\tau), \sigma_{iA}(t)] \rangle e^{i\Delta_p \tau} d\tau)$$

($i = C', B'$), which appear as absorption peaks at the detuning $\pm\Omega$ due to the positive values of $\bar{\rho}_{AA} - \bar{\rho}_{ii}$. On the contrary, as shown by the dotted and the dashed-dotted line, the terms of absorption spectrum related with the transitions $|j\rangle \to |A'\rangle$ ($j = B, C$) give rise to emission peaks locating at $\pm\Omega$, which can be attribute to the negative population differences between $|B\rangle$ and $|A'\rangle$, $|C\rangle$ and $|A'\rangle$, respectively. The superposition of the four spectral profiles is depicted by the short dashed line in Figure 7a. One can see that their total contributions to the whole spectrum is nearly zero. In Figure 7b the absorption profiles related with transition $|B\rangle \to |B'\rangle$, $|C\rangle \to |C'\rangle$ are plotted. It is seen that the absorption features show an order of magnitude of 10^{-4} due to the nearly equal populations between the dressed states involved in the transitions. For the cases of the absorptions in the transitions $|A\rangle \to |A'\rangle$, $|C\rangle \to |B'\rangle$ and $|B\rangle \to |C'\rangle$, the spectrum has an order of magnitude of 10^{-17}, and are not given here.

In Figure 7c we plot the sum contributions of the interference terms. Compare Figure 7c with Figure 3a, it is found that the contributions of interference terms constitute almost the whole absorption spectrum of the system. The main reason of this phenomenon can be concluded as: (1) the counteractive effects of the absorptions and the emissions of probe field in the transitions between $|A\rangle \to |B'\rangle$ ($|C'\rangle$), $|C\rangle$ ($|B\rangle$) $\to |A'\rangle$. (2) the nearly equal populations of $|B\rangle$ and $|C\rangle$.

For $p = 0.99$, according to equation (5), $G = G_0\sqrt{1-p^2} = 0.423$, which is much lower than the intensity of the dc field. Regard the pump field as a perturbation and consider the zeroth order approximation, the system have the same dressed states as those given in equation (13). This explains why the spectrum in Figure 3c is similar to that given in Figure 2c. However, for a nonzero pump field, the steady-state populations of $|B(\Omega_d)\rangle$ and $|C(\Omega_d)\rangle$ are also nonzero ($\bar{\rho}_{B(\Omega_d)B(\Omega_d)} = 0.003$ and $\bar{\rho}_{C(\Omega_d)C(\Omega_d)} = 0.013$), this influences the magnitudes of the two absorption resonances in Figure 3c compared with those in Figure 2c. The narrow-linewidth feature of the absorption resonance can serve as a signal for frequency stabilization for the coupling laser. In reference [17] an experiment is reported in a cascade system of Yb atoms in which an narrow electromagnetically induced transparency (EIA) spectrum is obtained and used to stabilize the frequency of the coupling laser. In their scheme the EIA spectrum is obtained by setting the probe detuning at one-photon resonance ($\Delta_p = 0$) and varying the detuning of the coupling field. Following the same procedure, we plot the spectrum obtained for our model in Figure 5. It can be seen that the absorption line can show narrow linewidth through adjusting the strength of the VIC, which provide a new way to perform the stabilization of the coupling laser frequency.

(3) $G_0 = 25$, $\Omega_d = 3$. Owing to the low intensity, the dc field can be regard as a perturbation to the system. The eigenstates of the system are identified by the Hamiltonian of the atom plus the pump field, which reads

$$\begin{aligned} |I\rangle &= |1, N\rangle, \\ |II\rangle &= \frac{1}{\sqrt{2}}[|2, N\rangle + |0, N+1\rangle], \\ |III\rangle &= \frac{1}{\sqrt{2}}[|2, N\rangle - |0, N+1\rangle], \end{aligned} \tag{21}$$

with energy

$$\begin{aligned} E_I &= (N+1)\hbar\omega, \\ E_{II} &= (N+1)\hbar\omega - \hbar G, \\ E_{III} &= (N+1)\hbar\omega + \hbar G. \end{aligned} \tag{22}$$

Import the Hamiltonian of perturbation

$$H_p = -(\hbar\Omega_d/\sqrt{2})[|I\rangle\langle II| + |I\rangle\langle III| + \text{H.c.}] \tag{23}$$

and utilize the standard perturbation approach [18], the eigenstates of the system to the first-order perturbation can be obtained as

$$\begin{aligned} |0(N, \Omega_d)\rangle &= \frac{1}{\sqrt{G^2+\Omega_d^2}}[G|1, N\rangle - \Omega_d|0, N+1\rangle], \\ |-(N, \Omega_d)\rangle &= \frac{1}{\sqrt{2G^2+\Omega_d^2}}[G|0, N+1\rangle + G|2, N\rangle \\ &\quad +\Omega_d|1, N\rangle], \\ |+(N, \Omega_d)\rangle &= \frac{1}{\sqrt{2G^2+\Omega_d^2}}[-G|0, N+1\rangle + G|2, N\rangle \\ &\quad -\Omega_d|1, N\rangle], \end{aligned} \tag{24}$$

with energy

$$\begin{aligned} E_{0(N,\Omega_d)} &= (N+1)\hbar\omega, \\ E_{-(N,\Omega_d)} &= (N+1)\hbar\omega - \hbar(G + \Omega_d^2/2G), \\ E_{+(N,\Omega_d)} &= (N+1)\hbar\omega + \hbar(G + \Omega_d^2/2G). \end{aligned} \tag{25}$$

The probe transition rate between two dressed states is proportional to the absolute square of the dipole matrix elements of the two states,

$$\begin{aligned} &|\langle 0(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p|0(N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{G^2\Omega_d^2}{(G^2+\Omega_d^2)^2}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle 0(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p| - (N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{G^4}{(2G^2+\Omega_d^2)(G^2+\Omega_d^2)}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle 0(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p| + (N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{G^4}{(2G^2+\Omega_d^2)(G^2+\Omega_d^2)}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle -(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p|0(N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{\Omega_d^4}{(G^2+\Omega^2)(2G^2+\Omega^2)}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle -(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p| - (N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{G^2\Omega_d^2}{(2G^2+\Omega_d^2)^2}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle -(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p| + (N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{G^2\Omega_d^2}{(2G^2+\Omega_d^2)^2}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle +(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p|0(N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{\Omega_d^4}{(G^2+\Omega_d^2)(2G^2+\Omega_d^2)}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle +(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p| - (N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{G^2\Omega_d^2}{(2G^2+\Omega_d^2)^2}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2, \\ &|\langle +(N+1, \Omega_d) \mid \boldsymbol{d}\cdot\boldsymbol{e}_p| + (N, \Omega_d)\rangle|^2 = \\ &\qquad \frac{G^2\Omega_d^2}{(2G^2+\Omega_d^2)^2}|\boldsymbol{d}_{10}\cdot\boldsymbol{e}_p|^2. \end{aligned} \tag{26}$$

Investigating the expressions (26) numerically, one found that the absorption of probe beam associated with the transitions $|+(N, \Omega_d)\rangle \rightarrow |0(N+1, \Omega_d)\rangle$ and $|-(N, \Omega_d)\rangle \rightarrow |0(N+1, \Omega_d)\rangle$ dominate in the nine transitions between adjacent manifolds because of the much higher values of the absolute square of the dipole matrix elements. These two transitions give rise to two absorption peaks located at $\Delta_p = -(G + \Omega_d^2/2G)$ and $\Delta_p = G + \Omega_d^2/2G$, respectively. For $p = 0,\ 0.45,\ 0.9$, the absolute values of the corresponding detunings are 25.18, 22.52 and 11.31, which coincide well with the graphical results. For the enhancement and the depression of the two different resonances, we can gain a physical insight by investigating the populations of the dressed states. In Figure 8 the

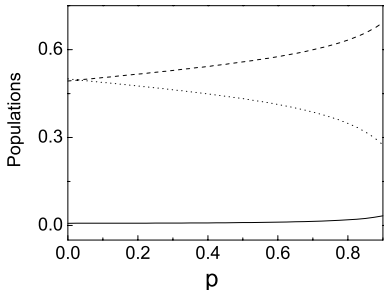

Fig. 8. Populations of dressed states as a function of p: ρ_{00} (solid line), ρ_{++} (dashed line), ρ_{--} (dotted line). Other parameters are the same as those in Figure 4.

evolutions of the the populations for dressed states versus p are depicted. It can be seen that the population ρ_{++} increase, ρ_{--} decrease, and ρ_{00} remains nearly unchanged with p increases. This phenomenon can be regarded as a quasi-trapping effect and $|+\rangle$ is the quasi-trapped-state for the reason that ρ_{++} can not be unity. The reduction of the probe absorption for the resonance at $\Delta_p = G + \Omega_d^2/2G$ and enhancement at $\Delta_p = -(G + \Omega_d^2/2G)$ is induced by the quasi-trapping effect which result in the decrease of the population difference $\rho_{--} - \rho_{00}$ and the increase of $\rho_{++} - \rho_{00}$ in comparison to the case with $p = 0$.

5 Conclusions

To sum up, we investigated the absorption of a weak probe in a degenerate V-type atom in the presence of a resonant pump field and a dc field. The effects of vacuum induced coherence (VIC) are considered. For different values of the pump field intensity, the absorption spectra are calculated. Physical interpretations of the spectra are given by introducing the dressed states and calculating the spectra in the dressed-state picture. For the case of zero pump field intensity, the dc-field coupling split the original absorption resonance into two symmetric components. When the VIC is taken into account, it is shown that it influences the linewidths and magnitudes of the two resonances in opposite ways. While in the case of equal intensities of the pump field and the dc field, it is interesting to find the probe absorptions in the transitions between dressed state have an counteractive effect, hence the absorption spectrum comes mostly from the absorptions induced by the interferences of the different transitions between dressed states. When the intensity of the pump field is much stronger than that of the dc field, it is pointed out that the VIC can lead to a quasi-trapping effect and result in the enhancement and the reduction of absorptions for different resonances.

Appendix: Bloch equations in the dressed-state picture

The equations of motion of the density matrix elements in the dressed-state picture can be obtained directly by projecting the master equation over the dressed-state basis. The master equation is given in equation (1) except that the Hamiltonian H takes the form given in equation (8). For the case of $G_0 = 0$ (considered in Fig. 2), the Hamiltonian is simplified and reads

$$H = \hbar\omega(\sigma_{11} + \sigma_{22}) - \hbar\Omega_d(\sigma_{12} + \sigma_{21}). \tag{27}$$

As an example, we give the equations of dressed-state basis shown in equation (10), which reads

$$\begin{aligned}
\dot{\rho}_{AA} &= [2c^3\gamma - 2b^2\gamma_1 - c^4\gamma_1 + c^2(2b^2\gamma_1 - \gamma_2)]\rho_{AA} + 4c^3\gamma\rho_{BB} + (1/\sqrt{2})[b(-1+2c^2)(\gamma + c\gamma_1)](\rho_{AB} + \rho_{BA}) \\
&\quad + (1/\sqrt{2})[b(-1+2c^2)(\gamma - c\gamma_1)](\rho_{AC} + \rho_{CA}) + (-c^4\gamma_1 + c^2\gamma_2)(\rho_{BC} + \rho_{CB}) + c^2(-2c\gamma + c^2\gamma_1 + \gamma_2), \\
\dot{\rho}_{BB} &= (b^2/2)(2c\gamma + 2b^2\gamma_1 - c^2\gamma_1 - \gamma_2)\rho_{AA} + [2(-1+b^2)c\gamma - c^2\gamma_1 - \gamma_2]\rho_{BB} + (1/\sqrt{2})[b(-1+2b^2)(\gamma + c\gamma_1)](\rho_{AB} + \rho_{BA}) \\
&\quad + (1/\sqrt{2})[b^3(\gamma - c\gamma_1)](\rho_{AC} + \rho_{CA}) - (1/2)(-1+b^2)(c^2\gamma_1 - \gamma_2)(\rho_{BC} + \rho_{CB}) + (b^2/2)(-2c\gamma + c^2\gamma_1 + \gamma_2), \\
\dot{\rho}_{AB} &= (-b/\sqrt{2})[\gamma + 2c^2\gamma + c(\gamma_1 + 2b^2\gamma_1 - c^2\gamma_1 - \gamma_2)]\rho_{AA} - (b/\sqrt{2})(\gamma + 4c^2\gamma + c\gamma_1)\rho_{BB} \\
&\quad + (1/2)(-2c\gamma - 2b^2c\gamma - 2b^2\gamma_1 - c^2\gamma_1 - 2b^2c^2\gamma_1 - \gamma_2 - 2i\Omega)\rho_{AB} - b^2c(\gamma + c\gamma_1)\rho_{BA} \\
&\quad + (1/2)[c^2\gamma_1 + 2b^2c(-\gamma + c\gamma_1) - \gamma_2]\rho_{AC} + b^2c(-\gamma + c\gamma_1)\rho_{CA} + (bc/\sqrt{2})(c^2\gamma_1 - \gamma_2)\rho_{BC} \\
&\quad + (b/\sqrt{2})[-\gamma + c(\gamma_1 + c^2\gamma_1 - \gamma_2)]\rho_{CB} - (bc/\sqrt{2})(-2c\gamma + c^2\gamma_1 + \gamma_2), \\
\dot{\rho}_{AC} &= (bc/\sqrt{2})(2c\gamma + 2b^2\gamma_1 - c^2\gamma_1 - \gamma_2)\rho_{AA} + (b/\sqrt{2})(\gamma + 4c^2\gamma - c\gamma_1)\rho_{BB} + [b^2c(\gamma + c\gamma_1) + (c^2\gamma_1 - \gamma_2)/2]\rho_{AB} \\
&\quad + b^2c(\gamma + c\gamma_1)\rho_{BA} + (1/2)[2(1+b^2)c\gamma - 2b^2\gamma_1 - c^2(\gamma_1 + 2b^2\gamma_1) - \gamma_2 + 2i\Omega]\rho_{AC} + b^2c(\gamma - c\gamma_1)\rho_{CA} \\
&\quad - (b/\sqrt{2})[\gamma + c(\gamma_1 + c^2\gamma_1 - \gamma_2)]\rho_{BC} + (bc/\sqrt{2})(-c^2\gamma_1 + \gamma_2)\rho_{CB} - (b/\sqrt{2})[\gamma + 2c^2\gamma - c(\gamma_1 + c^2\gamma_1 + \gamma_2)], \\
\dot{\rho}_{BC} &= (1/2)[-2b^4\gamma_1 - c^2\gamma_1 + \gamma_2 + b^2(-2c\gamma + c^2\gamma_1 + \gamma_2)]\rho_{AA} - 2b^2c\gamma\rho_{BB} - (b^3/\sqrt{2})(\gamma + c\gamma_1)\rho_{AB} \\
&\quad - (b/\sqrt{2})[(1+b^2)\gamma + (-1+b^2)c\gamma_1]\rho_{BA} - (b/\sqrt{2})(\gamma + b^2\gamma + c\gamma_1 - b^2c\gamma_1)\rho_{AC} + (b^3/\sqrt{2})(-\gamma + c\gamma_1)\rho_{CA} \\
&\quad + (1/2)[(-2+b^2)c^2\gamma_1 - (2+b^2)\gamma_2 + 4i\Omega]\rho_{BC} + (b^2/2)(c^2\gamma_1 - \gamma_2)\rho_{CB} - (1/2)[-c^2\gamma_1 + \gamma_2 + b^2(-2c\gamma + c^2\gamma_1 + \gamma_2)],
\end{aligned} \tag{28}$$

where the reduction operation

$$\rho_{I,J} = \sum_N \langle I(N,\Omega_d)|\rho|J(N,\Omega_d)\rangle, \quad I,J = A,B,C. \quad (29)$$

has been used. The density-matrix elements obey the conditions $\rho_{AA} + \rho_{BB} + \rho_{CC} = 1$ and $\rho_{IJ} = \rho^*_{JI}$.

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